Multi-Objective Fuzzy Project Portfolio Optimization with Divisibility: Integrating Financial Return, Downside Risk, and Societal Impact

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Abstract. This study develops a fuzzy multi-objective project portfolio selection model to support investment decision-making under uncertainty. The framework incorporates project divisibility and multi-period planning, enabling flexible allocation and scheduling of investments across competing projects. Investment costs and cash flows are represented as triangular fuzzy numbers to capture forecasting imprecision. The model simultaneously maximizes expected return while minimizing downside risk and societal impact. Expected return is evaluated using discounted cash flows, downside risk is measured through period-wise deviations below expected performance, and societal impact is assessed to support responsible investment decisions beyond purely financial criteria. A Zimmermann-based fuzzy linear programming approach is employed to obtain a compromise solution without predefined objective weights. By integrating uncertainty, project divisibility, and multi-period flexibility, the framework provides a robust decision-support tool for complex investment environments. Results show that project divisibility enhances portfolio flexibility and efficiency compared with non-divisible approaches. Partial project execution across multiple periods improves budget utilization, reduces risk concentration, and allows investments to be postponed or scaled as conditions change. Consequently, more projects can be included while satisfying financial and risk constraints, producing portfolios that better balance return, downside risk, and societal impact under uncertainty.

Keywords: *Project portfolio selection, fuzzy optimization, Zimmermann compromise programming, lower semi-variance index, disability-adjusted life years, project divisibility.*

1. INTRODUCTION

Project Portfolio Selection (PPS) is a strategic decision-making process that allocates limited resources among competing projects to maximize organizational performance. It plays a crucial role in aligning investment decisions with long-term objectives while operating under financial, operational, and environmental constraints (Ghasemzadeh & Archer, 2000).

Modern organizations face increasing uncertainty arising from imprecise financial forecasts, dynamic market conditions, and the need to balance multiple conflicting objectives. While traditional PPS models primarily emphasize financial returns, they often overlook risk exposure and societal consequences, limiting their applicability in complex decision environments (Mohagheghi et al., 2019).

To address financial uncertainty, fuzzy set theory has been widely adopted to represent investment costs and cash flows using triangular fuzzy numbers, incorporating pessimistic, most likely, and optimistic estimates (Tanaka et al., 2000). For risk assessment, this study employs the Lower Semi-Variance Index (LSVI), which focuses on downside deviations and better reflects risk-averse decision-making by capturing period-wise risk exposure.

Another limitation of conventional PPS models is their reliance on binary project selection, where projects are either fully accepted or rejected (Ghasemzadeh & Archer, 2000). In practice, organizations often require partial project implementation due to budget constraints and strategic considerations. Project divisibility therefore enables more efficient resource utilization and flexible multi-period scheduling (Arratia-Martinez et al., 2021).

Beyond financial and risk considerations, organizations are increasingly expected to account for societal impact in investment decisions. Consequently, this study incorporates societal impact as an additional objective, supporting more sustainable and socially responsible portfolio selection (Bihari et al., 2025).

2. LITERATURE REVIEW

Early PPS models relied on deterministic integer programming with crisp parameters (Ghasemzadeh et al., 1999; Masmoudi & Abdelaziz, 2018), which proved inadequate for dynamic investment environments where cash flows and costs were inherently uncertain. Tanaka et al. (2000) pioneered fuzzy portfolio evaluation by representing uncertain financial parameters as fuzzy numbers. Subsequent work by Srizongkhram et al. (2022) and Chiadamrong and Suthamanondh (2022) applied fuzzy multi-objective programming to simultaneously balance return and risk, demonstrating the advantage of fuzzy representations over crisp alternatives.

Downside-risk measurement attracted considerable research interest. Sarykalin et al. (2008) compared Value-at-Risk and Conditional Value-at-Risk, highlighting their limitations under non-normal distributions. LSVI emerged as a tractable downside-risk measure directly compatible with TFN-based defuzzification (Chiadamrong & Suthamanondh, 2022). For societal accountability, Murray and Lopez (1996) introduced DALY as a standardized health-impact metric and Bihari et al. (2025) extended its application to sustainable investment decision-making contexts.

Project divisibility was formally introduced and extended with interdependency considerations (Li et al., 2016). Arratia-Martinez et al. (2021) demonstrated significant efficiency gains from fractional resource allocation in portfolio scheduling. Zimmermann's (1978) fuzzy compromise programming has been widely adopted for multi-objective portfolio balancing, eliminating subjective weight assignment (Chiadamrong & Suthamanondh, 2022). No prior study simultaneously integrated fuzzy FNPV, LSVI, DALY, multi-period divisibility, and Zimmermann programming in a unified framework - the gap this paper addresses.

3. RESEARCH METHODS

3.1. Fuzzy Financial Parameters

Each project's cash inflows, investment capital, and setup cost are TFNs $\tilde{A} = (L, M, U)$, where L is the pessimistic estimate, U the optimistic estimate, and M the most likely value. The expected value is:

$$E\left[\widetilde{A}\right] = \frac{L + 2M + U}{4} \tag{1}$$

3.2. Three-Objective Model

The model simultaneously maximizes expected Fuzzy Net Present Value (FNPV), minimizes the Lower Semi-Variance Index (LSVI), and minimizes Disability-Adjusted Life Years (DALY) over T = 10 planning periods. For a TFN return $\tilde{R} = (L, M, U)$, LSVI captures downside deviation. The period-wise portfolio LSVI is:

$$\mathrm{LSVI}_t = \sum_{i=1}^{N} x_{i,t}.E\left(K_i\right).\mathrm{LSV}\left(\widetilde{R}_{i,t}\right) \tag{2}$$

where $x_{jt} \in [0,1]$ is the execution proportion of project j in year t (divisible) or {0,1} (non-divisible), $E[K_j]$ is the defuzzified investment capital, and LSV (·) computes the downside semi-variance of the TFN return. The three objectives are:

$$\mathrm{Max}(FNPV) = max\left(\left[\sum_{j\in J}\hat{C}_j\left(\sum_{t\in T}x_{jt}\cdot\frac{1}{(1+r)^t}\right)\right] - \left[\sum_{j\in J}\hat{I}_j\left(\sum_{t\in T}z_{jt}\cdot\frac{1}{(1+r)^t}\right)\right] - \left[\sum_{j\in J}\sum_{t\in T}\hat{S}_j.x_{jt}\cdot\frac{1}{(1+r)^t}\right]\right) \tag{3}$$

$$\min\left(\mathrm{LSVI}\right) = \min\left(\sum_{t=1}^{T}\mathrm{LSVI}_t.\frac{1}{\left(1+r\right)^t}\right) \tag{4}$$

$$min(DALY) = min(\sum_{i\in I}\sum_{t\in T}DALY_t.x_{it}) \tag{5}$$

where r = 10% is the discount rate, z_{jt} is the project start/restart indicator, and $DALY_j$ is the societal impact of project j.

3.3. Zimmermann Compromise Programming

To balance the conflicting objectives of maximizing profitability (FNPV) while minimizing downside risk (LSVI) and societal impact (DALY), a fuzzy multi-objective programming approach is employed. Using Zimmermann's max–min compromise method, the objectives are transformed into membership functions, and the model maximizes the minimum satisfaction level to obtain a balanced trade-off among financial performance, risk control, and societal sustainability under uncertainty.

Maximization of the expected fuzzy net present value

$$\max Z_1 = \mathrm{FNPV}(x, w) \tag{6}$$

This objective favors project portfolios that yield the highest expected financial performance after considering both investment and setup costs.

Minimization of the downside risk

$$\min Z_2 = \mathrm{LSVI}\ (x, w) \tag{7}$$

This objective promotes stability by penalizing portfolios that exhibit large or frequent negative deviations from the expected return.

Minimization of the societal impact

$$\min Z_3 = DALY\ (x, w) \tag{8}$$

This objective minimizes the societal burden of project portfolio execution by reducing total Disability-Adjusted Life Years (DALY). Lower DALY values reflect fewer adverse societal impacts, promoting socially sustainable project selection under uncertainty.

Together, Z_1, Z_2, and Z_3capture the trade-offs among financial performance, downside risk, and societal impact. As improving one objective may worsen another, the model seeks a balanced compromise solution that reflects organizational preferences through fuzzy satisfaction functions.

Let $FNPV^-$and $FNPV^+$denote the worst and best aspiration levels of FNPV obtained from single-objective maximization. As a benefit-type objective, higher FNPV values are preferred. Similarly, let $LSVI^-$and $LSVI^+$, and $DALY^-$and $DALY^+$, denote the best and worst aspiration levels of downside risk and societal impact, respectively, obtained from their corresponding single-objective minimization runs. Since LSVI and DALY are cost-type objectives, lower values are preferred. These aspiration levels are then used to construct the corresponding linear membership functions.

$$\mu_1(\mathrm{FNPV}) = \begin{cases} 0, FNPV \ \le \mathrm{FNPV}^- \\ \frac{FNPV - \ \mathrm{FNPV}^-}{\mathrm{FNPV}^+ - \ \mathrm{FNPV}^-}, \mathrm{FNPV}^- < FNPV < \mathrm{FNPV}^+ \\ 1, FNPV \ \ge \mathrm{FNPV}^+ \end{cases} \tag{9}$$

$$\mu_2(\mathrm{LSVI}) = \begin{cases} 1,\ LSVI \ \ge \mathrm{LSVI}^- \\ \frac{LSVI - \ \mathrm{LSVI}^-}{\mathrm{LSVI}^+ - \ \mathrm{LSVI}^-}, \mathrm{LSVI}^- > LSVI > \mathrm{LSVI}^+ \\ 0, LSVI \ \le \mathrm{LSVI}^+ \end{cases} \tag{10}$$

$$\mu_3(\mathrm{DALY}) = \begin{cases} 1,\ DALY \ \ge \mathrm{DALY}^- \\ \frac{DALY - \ \mathrm{DALY}^-}{\mathrm{DALY}^+ - \ \mathrm{DALY}^-}, \mathrm{DALY}^- > DALY > \mathrm{DALY}^+ \\ 0, DALY \ \le \mathrm{DALY}^+ \end{cases} \tag{11}$$

Here, μ_1, μ_2, and $\mu_3$denote the satisfaction levels for portfolio value, downside risk, and societal impact, respectively. Each membership function normalizes the corresponding objective value to a scale of 0 to 1, where higher values indicate greater satisfaction.

The fuzzy compromise model is formulated as a max–min problem that seeks to maximize the lowest satisfaction level across objectives:

max λ

subject to

$$\mu_1(\mathrm{FNPV}) \ge \lambda,\ \mu_2(\mathrm{LSVI}) \ge \lambda,\ \mu_3(\mathrm{DALY}) \ge \lambda \tag{12}$$

This formulation maximizes λ, ensuring that all objectives achieve at least the same satisfaction level. The resulting solution provides a balanced trade-off between profitability and stability under uncertainty.

4. RESULTS AND DISCUSSION

4.1. Case Study Setting

Ten projects (P1-P10) are evaluated over a ten-year horizon. Table 1 summarizes fuzzy cash flows, investment capital, setup costs, lifespans, and DALY values. Global constraints: total budget ≤ USD 1,650 M; annual spending ≤ USD 250 M; minimum total investment ≥ USD 520 M; LSVI cap ≤ 85 M USD/year; execution proportion α ∈ [0.2, 1.0].

Table 1. Fuzzy project data and model parameters

Project	Cash Flow (USD million) (L, M, U)	Investment (USD million) (L, M, U)	Setup (USD million) (L, M, U)	Life (yr)	Societal Impact (DALY/yr)
P1	(50, 70, 90)	(100, 120, 150)	(10, 20, 30)	4	(0.005, 0.008, 0.012)
P2	(50, 80, 110)	(120, 140, 170)	(10, 20, 30)	5	(0.008, 0.012, 0.018)
P3	(20, 60, 120)	(80, 100, 130)	(10, 20, 30)	3	(0.004, 0.006, 0.009)
P4	(50, 100, 180)	(140, 170, 200)	(10, 20, 30)	6	(0.016, 0.025, 0.038)
P5	(40, 90, 140)	(130, 160, 190)	(10, 20, 30)	4	(0.007, 0.010, 0.015)
P6	(30, 60, 90)	(60, 110, 160)	(10, 20, 30)	5	(0.007, 0.011, 0.016)
P7	(90, 110, 140)	(110, 130, 160)	(10, 20, 30)	3	(0.006, 0.009, 0.013)
P8	(80, 150, 220)	(160, 200, 240)	(10, 20, 30)	4	(0.013, 0.020, 0.030)
P9	(90, 220, 350)	(120, 150, 180)	(10, 20, 30)	5	(0.020, 0.030, 0.045)
P10	(60, 120, 180)	(140, 170, 210)	(10, 20, 30)	4	(0.010, 0.015, 0.022)

4.2. Zimmermann Compromise Solution

Table 2 presents the optimal execution schedules under Zimmermann's compromise model. Under non-divisible execution, four projects (P2, P3, P7, P9) were selected with binary scheduling - P7 in Y1-Y3, P2 in Y4-Y8, P9 in Y5-Y9, and P3 in Y8-Y10 — with temporal separation applied to control peak-year LSVI and DALY exposure. Under divisible execution, five projects (P1, P2, P6, P7, P9) participated fractionally across the horizon, with P6 postponed in Y7 due to no new investment in that period. This phased fractional entry enabled five projects to coexist within the same annual budget and LSVI constraints that restricted the non-divisible portfolio to four.

Remarks: * denotes a postponement of project execution in the specified years. The values reported in each year represent the cumulative execution proportion of the corresponding project up to that planning period. A value of 1 indicates that the project has been fully invested, while the fractional value indicates partial execution accumulated over time.

Table 3 quantifies the performance difference. Divisible execution generated USD 137.38 M more in expected NPV (+18.5%), while LSVI rose modestly by USD 26.86 M (+8.5%) and DALY/yr by 0.039 (+15.0%). The satisfaction level λ* improved from 0.5118 to 0.6153 (+20.2%), indicating that the proportional FNPV gain substantially outpaced the marginal increases in risk and societal burden. The rise in λ* is the decisive indicator: under Zimmermann's framework, it reflects the worst-case satisfaction across all three objectives simultaneously, confirming that divisible execution achieved a proportionally superior multi-objective balance.

5. CONCLUSION

This study proposed a fuzzy multi-objective project portfolio optimization model that simultaneously considers financial performance, downside risk, and societal impact under uncertainty. By integrating fuzzy set theory, downside-risk assessment, DALY-based societal evaluation, and divisible multi-period scheduling, the framework extends conventional project portfolio selection models toward more adaptive, sustainable, and risk-aware investment decision-making. Its applicability depends on the feasibility of divisible project execution and the validity of the underlying financial and societal impact estimates.

The results demonstrate that project divisibility enhances portfolio flexibility, resource utilization, and downside-risk control. Partial project execution and postponement enable more efficient allocation of investments while reducing concentrated exposure to financial uncertainty and societal burden. The inclusion of DALY further broadens portfolio evaluation beyond financial criteria by incorporating societal and public-health considerations. In addition, fuzzy numbers, period-wise LSVI, and Zimmermann's fuzzy multi-objective programming approach effectively capture uncertainty and generate balanced compromise solutions among profitability, risk mitigation, and societal sustainability without requiring predefined objective weights.

Overall, the proposed framework provides a comprehensive foundation for developing resilient, flexible, and socially responsible project portfolios under uncertainty.

Future research may extend the model by incorporating additional resource constraints, inter-project dependencies, and broader sustainability measures such as environmental or ESG indicators. Further improvements may include adaptive uncertainty-updating mechanisms, metaheuristic optimization techniques for large-scale problems, and empirical applications in industrial, healthcare, infrastructure, and public-sector portfolios to validate the model's practical effectiveness.

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